Math 10B with Professor Stankova
Worksheet, Discussion #26; Thursday, 4/26/2018
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1 Matrices and Eigenvalue Review

1.1 Example

1. Find the general solution to

$$\begin{cases} y_1'(t) = 3y_1(t) + 5y_2(t) \\ y_2'(t) = -y_1(t) + y_2(t) \end{cases}$$

2. Represent the second order linear homogeneous differential equation $y'' - 4y' + 3y = 0$ as a system of linear differential equations using $y_1(t) = y$ and $y_2(t) = y'(t)$.

1.2 Problems

3. True False If $A\vec{v} = 3\vec{v}$, then $A^{100}\vec{v} = 3^{100}\vec{v}$.

4. True False When we write a second order homogeneous linear differential equation as a system of first order equations and solve for y_1, y_2, then whatever we get for y_2 will always be the derivative of whatever we get for y_1.

5. True False If $\vec{x}, \vec{y}$ are two solutions to $\vec{z}' = A\vec{z}$, then any linear combination of them is also a solution.

6. True False If λ is an eigenvalue for A, then there are infinitely many solutions to $A\vec{v} = \lambda\vec{v}$.

7. Find the general solution to

$$\begin{cases} y_1'(t) = 2y_1(t) - 2y_2(t) \\ y_2'(t) = y_1(t) + 4y_2(t) \end{cases}$$

8. Find the general solution to

$$\begin{cases} y_1'(t) = y_1(t) - 2y_2(t) \\ y_2'(t) = 2y_1(t) + y_2(t) \end{cases}.$$

9. Find the characteristic equation of the matrix of a system of linear differential equations if one solution is $\begin{pmatrix} 2e^{2t}\cos(2t) \\ 4e^{2t}\cos(2t) - 3e^{2t}\sin(2t) \end{pmatrix}$

10. What are the eigenvalues and eigenvectors of the matrix of a system of linear differential equations if one solution is $\begin{pmatrix} 3e^{2t} + 5e^{4t} \\ e^{2t} - e^{4t} \end{pmatrix}$.

11. Find the general solution to $y'' + 3y' - 10y = 0$ by writing it as a system of first order differential equations.

12. Find the general solution to $y'' + y' - 12y = 0$ by writing it as a system of first order differential equations.

13. Name 3 reasons why we care about eigenvalues and eigenvectors.

2 Linear Regression

2.1 Concepts

14. Often when given data points, we want to find the line of best fit through them. To them, we want to approximate them with a line $y = ax + b$. We represent this as a solution where we want to solve for a, b. In matrix vector form and data points (x_i, y_i), this is represented as

$$A\vec{x} = \vec{b} \rightarrow \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}.$$

Often, we cannot find a perfect fit (if not all the points lie on the same line). So we want to find the error. One way to find the error is to take the least square error or $E = \sum (y_i - (ax_i + b))^2$, the sum of the squares of the error. The choice of a, b that minimizes this is

$$\begin{pmatrix} a \\ b \end{pmatrix} = (A^T A)^{-1} A^T \vec{b}.$$

2.2 Example

15. The number of people applying to Berkeley is given in the following table:

Year	2011	2012	2013	2014	2015	2016	2017
Applicants(in 1000s)	53	62	68	74	79	83	85

Predict how many people will apply this year.

2.3 Problems

16. True False The matrix $A^T A$ will always be square.

17. Consider the set of points $\{(-2,-1),(1,1),(3,2)\}$. Calculate the line of best fit.
18. Find the line of best fit and the error of the fit of the points $\{(-1,2),(0,-1),(1,1),(3,2)\}$ and use it to estimate the value at 2.
19. Consider the set of points $\{(-2,-1),(1,1),(3,2)\}$. Calculate the square error if we estimate it using the line $y=x$. Then calculate the square error if we use the line $y=0$. Which is a better approximation?